

# Case of Explosive Wing-Flaperon Flutter

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**Nonlinearities in aircraft structures can often affect flutter, and usually cause negative feedback resulting in limit-cycle oscillation. Positive feedback is rarely observed, and little studied, but can be very dangerous (one author has called it “explosive flutter”). This paper presents a case of an aircraft which has had occasional low-speed (around 100 knots) wing-flaperon flutter in service. Evidence is produced from a ground vibration test of the flaperon control circuit, and flutter analysis, to show that strong positive feedback is present and this explains the occasional events. Significant initial excitation amplitude is required before the system becomes unstable and damaging flutter results. This paper suggests that this episode provides some lessons for general aviation aircraft design and certification, where manually operated mass balanced control surfaces are common.**

## Introduction

THERE has been much study of nonlinear aeroelasticity recently [1,2], and many instances of its occurrence [3] (mainly transonic). However, virtually all instances and studies have been of the phenomenon of limit-cycle oscillation (LCO), so much so that LCO is often considered synonymous with nonlinear flutter [1,2]. However, LCO is only one side of nonlinearity, the stable negative feedback case where amplitude settles down to a steady-state value. The other possibility, positive feedback, has received little attention and (fortunately) incidents seem to be rare.

Thomas et al [2] divide LCO into soft flutter or benign LCO, and hard flutter or explosive LCO. Figure 1 indicates how flutter speed (when damping drops to zero) changes with amplitude. The second type is initially unstable, but has a knee so flutter settles to a steady-state LCO. Dessi and Mastroddi [4] include a third case, without the knee, calling it explosive flutter or destructive flutter. The term LCO is not applicable to this case; if there is sufficient excitation to reach the stability boundary below the linear flutter speed, amplitude will grow without limit until destruction occurs. They go on to say “such analysis is not used in the actual design of aircraft,” but that experimental evidence supports cases of the second type of nonlinear behavior, with a stable LCO. The most usual structural nonlinearities in theoretical models, backlash and hardening springs, also usually yield LCO.

This paper describes the investigation of incidents on an aircraft which experiments and analysis point to being a case of explosive flutter (the third type). It is unusual for an aircraft to have a flutter problem in the flaps-down configuration with speed restricted to about 100 kn. It is also unusual for this to occur only rarely, not showing up in flight flutter tests, and yet cause structural damage when it does happen. The story of this problem is an interesting lesson in the pitfalls of control system design, flutter analysis, and testing in an aircraft of unusual configuration.

## Background

The Nomad is a twin-turboprop FAR 23 certificated aircraft of up to 4200 kg (all up weight) with a strutted high wing. Within its class, its wing control surfaces are unique: full-span double-slotted flaps plus a spoiler (Fig. 2). The flaps are split into inboard and outboard halves, the outboard rear flap also serving as an aileron, i.e., a

“flaperon.” In cruise configuration, the aileron has maximum travel and the spoiler is inoperative. The aft flaps can be extended to 10, 20, or 38 deg, and landing is normally at 38 deg, as shown in Fig. 3. In this configuration, the outboard flaperon has reduced travel as an aileron, and the spoiler operates. The flaperon is hinged well below the chord line (dictated by flap movement geometry) and has a pendulous mass balance at the tip (shown in Fig. 4). Aileron control is entirely manual (i.e., powered by pilot effort), with a cable quadrant in each wing. Flap angle is controlled by an electrical actuator, with pushrods going along the rear spar.

The original flutter certification to FAR 23.629 was by analysis and flight test, but only in cruise configuration (0 deg flap). No need for flaps-down flutter justification was perceived. A ground vibration test (GVT) of the entire aircraft was also conducted at 0 deg flap, identifying natural airframe modes and the flaperon rotation mode at about 10–11 Hz [5].

In over 20 years of service, there have been about 14 reports of flaperon damage on landing, of varying severity with flaperon tip separation and wing primary structure buckling in the worst cases. All incidents occurred at 38 deg flap. Also of note were repeated incidents on the same aircraft; three aircraft had two incidents each, and one aircraft had three incidents (each time after replacement of a damaged flaperon).

It is instructive to look at the approaches made to solving the problem. After the first two incidents, both in situations of high turbulence, it was concluded to be a static overload due to combined high-speed, head-on gust and control deflection. Although it was very hard to come up with a load case that could theoretically break the flaperon, this explanation was accepted by the manufacturer and the Civil Aviation Safety Authority (CASA, the Australian airworthiness authority), and a strengthening modification was designed. Further incidents to aircraft not yet so modified were dismissed.

Then came the first incident to a modified aircraft. This led CASA to impose a restriction prohibiting the use of 38 deg flap. By this time, it had been recognized by all parties that flutter was involved, owing to reports of vibration felt in the aircraft and seen by witnesses in the earlier incidents, but the view was still held that it resulted from a preexisting defect, perhaps due to static overload. Anecdotes of flap speed limit abuse by pilots reinforced this opinion.

Resolution of this situation was provided by CASA imposing a special requirement on the Nomad: “Flutter clearance to FAR 23.629 with flaps down, to a speed of  $V_{FE}/0.9$ ” [ $V_{FE}$  is maximum permitted speed with flaps extended: note that FAR 23.629 requires clearance to  $V_D$  (design dive speed), whereas  $V_{NE}$  (never-exceed speed) is set to  $0.9V_D$  by FAR 23.1505]. When compliance with this is demonstrated, the restrictions would be lifted. Flight flutter tests were conducted, using atmospheric turbulence as the primary means of excitation (as had been done in the original flaps-up certification),

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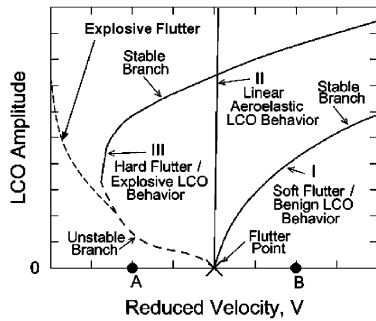


Fig. 1 Types of nonlinear flutter (Thomas et al. [2] and Dessi and Mastroddi [4]).

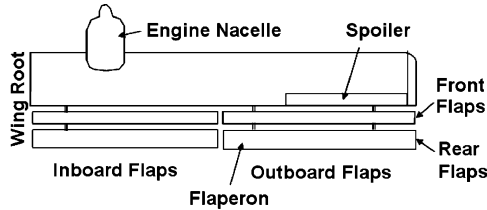


Fig. 2 Nomad wing planform showing flap geometry.

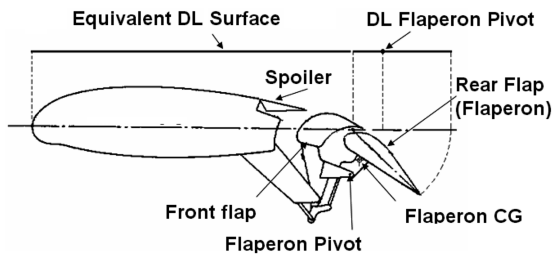


Fig. 3 Wing section, flaps at 38 deg, showing equivalent doublet lattice (DL) surface.

and adequate damping was observed. The restriction on 38 deg flap was then lifted.

Then there was another incident, this time from an operator with a good history of maintenance and piloting. At this point, the author came to the opinion that the aircraft is probably prone to flutter without any preexisting defects; it is something inherent in the design. However, it only happened occasionally and it was not clear what set it off. Recent flight flutter tests had not reproduced it. Further investigation was necessary.

The approach taken, as described in the rest of this paper, was 1) modeling and simple analysis of flutter in the flaps-down configuration, indicating the modes most likely to be the cause, 2) a GVT to measure the flaperon rotation mode shape and study how it changes with amplitude, and 3) more refined flutter analysis using this measured data.

Then follows a discussion of how the incidents may have occurred, why flight flutter testing did not reveal the problem, and lessons learned relevant to general aviation (i.e. FAR 23) aircraft.

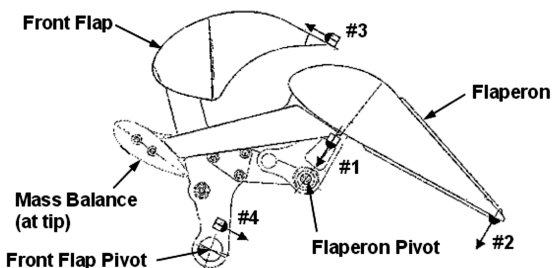


Fig. 4 Accelerometers on flaperon and front flap.

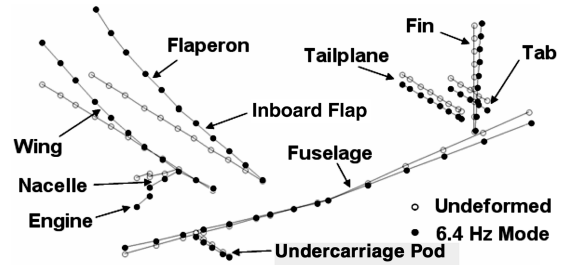


Fig. 5 Structural FEM model, showing predicted 6.4 Hz mode.

## Flutter Mechanism

Classical flutter is a coupling of two (or more) natural vibration modes. With wing-aileron flutter, it is usually a coupling of aileron rotation and a wing bending mode. It is normally analyzed with a finite element method (FEM) model representing the inertia, stiffness, and aerodynamics of the aircraft. Such a model had been used for analysis in the cruise configuration. The wings and other major components were modeled as beams (see Fig. 5), and the aerodynamics was from a superimposed doublet lattice grid [6]. Natural vibration modes of the beam structure were computed, and these were used as generalized degrees of freedom for the complete flutter model. Figure 5 shows the computed first wing bending mode at 6.4 Hz, very similar to the 6.5 Hz mode observed in the original ground vibration test. This model had never been used for other flap deflections, because 1) it was originally not thought necessary; flutter was a high-speed phenomenon; 2) the aerodynamic tool available, doublet lattice method, can only analyze a flat streamwise surface; and 3) the large separated flow region (observed with tufts in flight test) would make any linear aerodynamics very inaccurate.

However, Pines ([7], reviewed in [8]) had shown that the behavior of classical flutter can be accurately predicted even with very approximate aerodynamics. The primary requirements for flutter are frequency proximity of the two natural modes, and an adverse mass coupling. The latter is the off-diagonal term in the generalized inertia matrix defined as

$$IM_{ij} = \sum_k m_k x_{ik} x_{jk} \quad (1)$$

where  $m_k$  are the mass elements, and  $x_{ik}$  and  $x_{jk}$  the displacement in modes  $i$  and  $j$ . It is considered adverse if it is positive, when the modes are in the sense of trailing edge up (the AGARD convention [9]). For control surfaces, this effectively means they should be balanced or leading-edge heavy.

It was easy to modify the dynamics of this model to accommodate other flap angles. The existing inboard flap and flaperon beams were rotated and translated to the new position, based on nominal geometry (due to "blowback," the actual angle in flight is less than nominal by some degrees, but the results are indicative). This model was then used to compute the mass coupling terms between flaperon rotation and the various airframe natural modes. The results are shown in Table 1. It can be seen that with the 6.4 Hz mode it is negative (stable) for zero and 20 flap, but goes positive (unstable) at 38 deg flap. No other aircraft natural modes had this property (note that the absolute value of the terms is arbitrary, as it depends on modal amplitude; what is important is relative magnitude and sign). The reason for the change in mass coupling with flap angle is easily seen in Fig. 3. With the c.g. well above the hinge line (even with the pendulous mass balance), it moves aft with increasing flap angle.

This clearly implicates the first wing bending mode, however, the frequency at 6.5 Hz is well separated from the 10–11 Hz flaperon frequency. More investigation was required.

## Flaperon Control Circuit Measurements

The flap and aileron controls are mixed through an "interchange mechanism," whereby flaperon movement range from the control wheel is reduced as flap angle is increased. This tortuous path of bellcranks and pushrods from the control cable quadrant to the

**Table 1** Mass coupling terms, flaperon rotation to various other modes

Flap angle	Mode								
	6.4 Hz	9.0 Hz	10.0 Hz	13.0 Hz	14.3 Hz	16.2 Hz	18.7 Hz	21 Hz	24 Hz
0	-0.00414	-0.0056	-0.0003	-0.0018	-0.0021	-0.0029	-0.0042	0.0122	-0.0038
20	-0.00105	-0.0035	-0.0003	-0.0020	-0.0020	-0.0031	-0.0044	0.0127	-0.0044
38	0.00205	-0.0012	-0.0003	-0.0021	-0.0018	-0.0032	-0.0044	0.0133	-0.0046

flaperon was flexible just under hand pressure, so could interact with flaperon rotation. A GVT was set up to study this, as shown in Fig. 6. A Nomad wing was set up in a rig supported so as to put the wing's own natural mode frequencies above the range of interest. The aileron cables were terminated at the wing root, clamped to a fixture (the cable tension was set by a hanging weight before clamping). The flaperon was excited by an electromagnetic shaker, with load cell, at the inboard hinge, where the roll control actuation is also located. To simulate airload, which is always present when the flap is deflected, a bungee cord was attached to the trailing edge. To compensate for the stiffness of the bungee cord, an extra tuning mass was attached to the flaperon trailing edge, and adjusted for the observed frequency.

A total of 13 accelerometers were placed on all flap surfaces. At the inboard and outboard hinge stations of the outboard flaps, there are four accelerometers, as shown in Fig. 4. The forward and aft flaps each have 3 degrees of freedom, totaling six, but the connection by the flaperon hinge reduces this to four. The four accelerometers provide sufficient information to determine the complete motion of the forward and aft flaps in the wing section plane. At the flaperon tip, where the mass balance is located, there were two accelerometers (horizontal and vertical) on the mass balance and one on the flaperon trailing edge.

Constant force sine wave sweep excitation was applied, from 4 to 11 Hz at 0.15 Hz/s, with closed loop force control. Runs were conducted at force levels varying from 5 to 160 N peak. Data was processed using the Signal Star (version 4.3b) software from Data Physics to produce acceleration amplitude  $g$  and transfer function  $g/N$  frequency responses. In all cases, a single resonance was observed, this being primarily flaperon rotation.

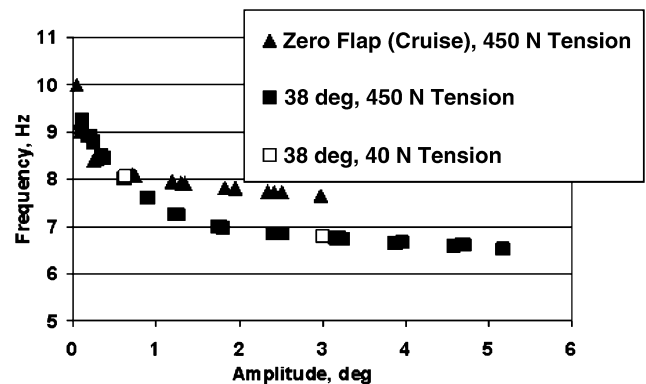
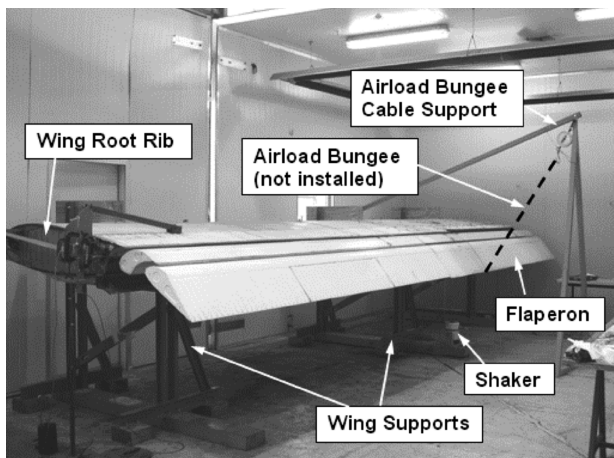
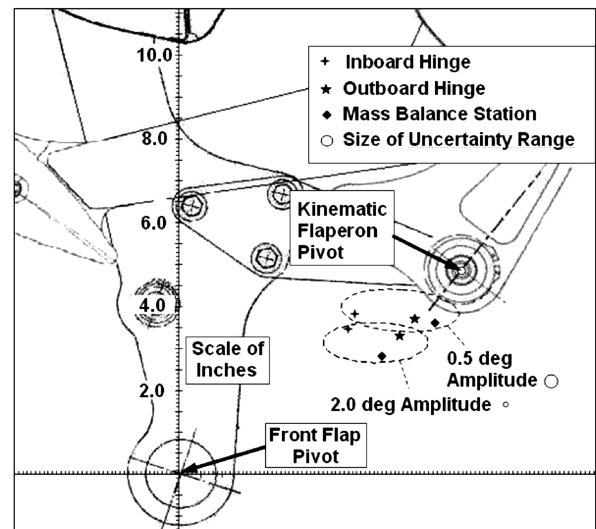
The frequency of the flaperon mode at different amplitudes for both zero and 38 deg flap settings are shown in Fig. 7. Here the ordinate is the flaperon rotation angle amplitude, computed from the two accelerometers on the flaperon. Only at the lowest amplitude is the frequency at zero flap consistent with the original aircraft GVT (10–11 Hz). At 38 deg flap, the frequency rapidly drops to close to the 6.5 Hz first wing bending frequency, supplying the missing flutter requirement of frequency proximity [it may also be noted that aileron cable tension (nominally 450 N, only 40 and 450 N results shown on graph) had no noticeable effect on the frequency].

The relative transfer functions at each of the three wing stations were used to compute the actual centers of rotation (COR) of the flaperon. Figure 8 shows the CORs are not at the theoretical flaperon

pivot, but well forward, and move forward with increasing amplitude (circles showing uncertainty range are based on noise in the transfer function). The reason for this is increasing motion of the forward flap. This is supposed to be fixed for a given flap setting, but there is sufficient flexibility in the flap control system to allow it to move, and the flaperon pivot is on the forward flap. It can easily be seen that if flaperon motion is a combination of forward flap rotation and flaperon rotation, then the true COR will be somewhere on a line between the two theoretical pivots, which is approximately borne out in Fig. 8. This forward movement of the COR with amplitude reduces the effective mass balance and increases adverse mass coupling. Figure 9 shows the computed mass coupling for the measured flaperon modes with the 6.4 Hz wing bending mode. The positive (adverse) mass coupling gets steadily worse with amplitude (the value at zero amplitude assumes the COR at the theoretical pivot; below 0.5 deg the measured COR is too uncertain).

### Flutter Analysis

Some flutter analysis was done using the modified Nomad FEM model. As stated, it was easy to modify the dynamics of this model to

**Fig. 7** Accelerometers on flaperon and front flap.**Fig. 6** Flaperon ground vibration test rig.**Fig. 8** Centers of rotation of flaperon at 38 deg for various spanwise locations.

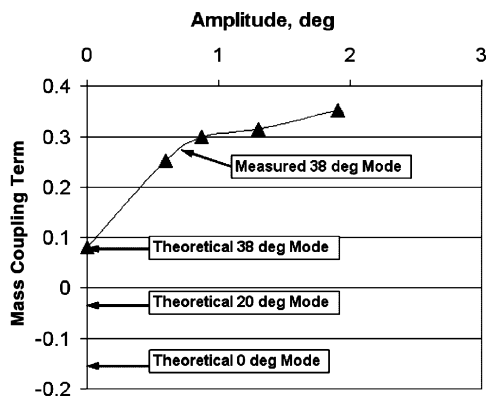
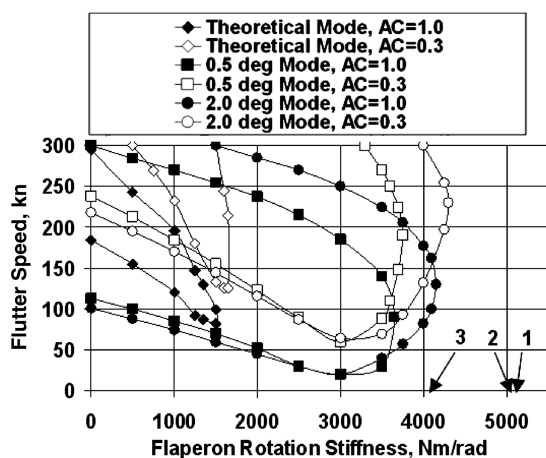


Fig. 9 Mass coupling terms, flaperon rotation to 6.4 Hz wing bending mode.



Stiffness Computed from:

- 1) Original GVT Frequency (11 Hz) and Theoretical Mode
- 2) Measured 0.5 deg Amplitude Mode (8 Hz)
- 3) Measured 2.0 deg Amplitude Mode (7 Hz)

Fig. 10 Flutter boundaries at 38 deg flap for various parameters.

accommodate other flap angles. For the aerodynamics, the wing was imagined flattened out (see Fig. 3), and displacements on the flap portions were based on flap displacements multiplied by the cosine of the flap angle.

The flutter equation was solved by the k-method [10] with no structural damping. The following flaperon parameters were varied: 1) rotation stiffness, 2) amplitude 0 (theoretical mode), 0.5, and 2 deg, and 3) aerodynamic coefficient (AC) 0.3 and 1.0. This is a coefficient on the pressure resulting from aileron deflection. It is commonly known that trailing-edge surfaces have less than theoretical effectiveness, by as much as 50% [11]. For the Nomad flaperon, this was estimated by comparing the aircraft roll rate due to flaperon deflection with that predicted by the antisymmetric doublet lattice model at zero frequency parameter (steady-state). This gave a rather low value of 0.35, which may be due to the unusual geometry with low pivot position.

At zero and 20 deg flap, no flutter below 300 kn was observed. Figure 10 shows the results for 38 deg. It can be seen that the minimum stiffness to prevent low-speed flutter gets higher with amplitude. Figure 11 shows the velocity, growth, and frequency (V-G-F) plot for a critical case. The fluttering mode at 60 kn and 6 Hz is a coupling of flaperon rotation and first wing bending.

The actual rotation stiffness was estimated as follows. Using the measured transfer functions, and the masses and moments of inertia of the flaperon and other flaps, the generalized mass of the flaperon rotation mode (the diagonal inertia matrix entry  $IM_{33}$ ) was computed. Combined with the measured frequency  $\omega_3$ , the rotation

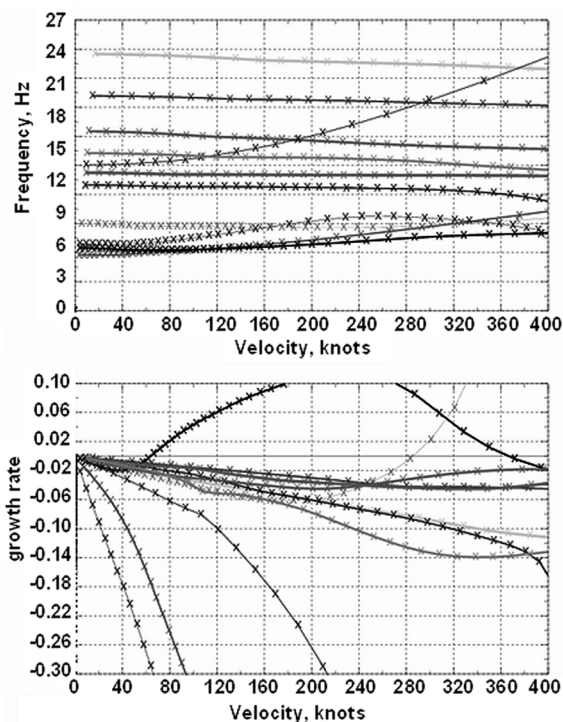


Fig. 11 V-G-F plot for case of 38 deg flap, 2 deg amplitude, 3000 Nm/rad stiffness, and AC = 0.3.

stiffness  $K_3$  for the flaperon was found by

$$K_3 = IM_{33}\omega_3^2 \quad (2)$$

The values for 0.5, 2.0 deg, and the original GVT result (assuming the flaperon was rotating about its theoretical pivot) are indicated in Fig. 10. This shows stiffness in fact changes very little initially; most of the frequency drop is due to the increased inertia resulting from the change of COR. It can be seen that at 2 deg amplitude the computed stiffness is very close to the onset of flutter.

### Implications: Possible Scenario

A possible scenario for the cause of the incidents on the Nomad was constructed:

1) Most of the time, the flaperon is stable at 38 deg. The mass coupling is slightly positive, but the frequency for very small amplitudes is 10–11 Hz, well above the wing bending frequency.

2) As a result of a gust or some other disturbance, a large amplitude vibration is excited. The mass coupling increases and the frequency drops closer to the 6.5 Hz wing bending frequency. Whether the damping is positive or negative will depend on the specific characteristics of the disturbance and the aircraft. The flaperon and wing bending frequencies will vary slightly from aircraft to aircraft (This may explain repeated incidents on the same aircraft).

3) If the amplitude growth rate is even slightly positive, the amplitude will grow, increasing the growth rate, and explosive flutter results.

### Level of Excitation

We might ask what level of excitation we might expect for this aircraft. Consider first the recent flight flutter tests, which used atmospheric turbulence as the excitation. This is of course highly variable, but some accelerometer records from these tests were double integrated to estimate angular displacement. The results did not exceed  $\pm 0.25$  deg. From Figs. 8 and 9, it can be seen this is still far from the worst nonlinear effects. It is not surprising that no flutter or seriously reduced damping was observed.

What excitation might it be subjected to in service? An obvious source is gusts. From static measurement of the flap circuit stiffness,

it was calculated that a 25 ft/s head-on gust would blow the flaps back 1.5 deg (trailing-edge devices are more affected by head-on rather than normal gusts). At that amplitude, all the nonlinear effects are close to their asymptotic values, so explosive flutter is likely. This of course relies on the gust being sharp enough to excite a 6 Hz mode; that means a length scale of about 2 m at 100 kn. No study of the likelihood of such a gust has been conducted, but it is clearly a possibility and could be responsible for the known incidents.

### Flight Flutter Tests

The current FAR 23.629 requires flight flutter tests (FFT) to demonstrate freedom from flutter (for original certification). In the light of this experience, the wisdom of reliance on FFT may be questioned. FFT were conducted, with no result, and yet there was subsequently another incident. Two reasons can be identified here.

#### Excitation Amplitude

FAR 23.629 requires “proper and adequate attempts to induce flutter.” The advisory circular [12] suggests various excitation methods (it does not include atmospheric turbulence) and reiterates the important criterion of adequate excitation, without defining it. The author suggests two possible meanings:

1) Equal to the greatest excitation expected in service. Databases of turbulence and gusts are generally statistical and of use in fatigue where damage accumulates. Flutter, however, is a single incident failure, like static load. The author suggests the airworthiness code should specify a “limit” excitation, as it does for static loads.

2) Sufficient to span the nonlinearities in all relevant parameters and approach the asymptote for unlimited amplitude. This makes more sense with the current state of knowledge, as at least some of these nonlinearities can be measured in ground testing. After all, amplitude is of importance only as a result of nonlinearity; if all was linear, “adequate excitation” would not be an issue.

In the latter case, results of a GVT should be plotted against amplitude, and excitation level chosen on that basis (with a sanity check that it is not obviously greater than could occur in service). For the Nomad, this would imply atmospheric turbulence was probably not sufficient.

#### Critical Cases

It is obviously impossible to test every possible flight condition and every individual aircraft variation to be sure they are all free from flutter. The normal procedure is to test critical cases. But what are these? Here, one normally relies on theory: parametric variations are made in runs of a computer model to find the critical conditions for that model. Then it is assumed the critical conditions for the real aircraft will be the same, so only those are tested. This assumption is rarely questioned, but it is at the heart of the entire concept of testing.

Consider the case where the flutter speed has a narrow valley at a certain value of a parameter, such as rotation stiffness for many of the curves in Fig. 10. Where this valley occurs in reality could be influenced by many things, and could vary between individual aircraft. Then there is the problem of actually setting up a chosen flaperon stiffness, particularly when it varies with amplitude. It is extremely difficult to test this critical case. This problem can apply to all classes of aircraft, but is more pressing for FAR 23 standard than transport aircraft, as flight tests are generally more limited by financial constraints.

### Lessons Learned and Relevance to General Aviation Aircraft Certification

In the author's view, this episode provides some lessons for the design and certification of general aviation (i.e., FAR 23) aircraft

with manual (i.e., unpowered) controls. Firstly, it is a good idea to keep to conventionally hinged fully mass balanced ailerons. If you must have other geometry

1) Flutter can occur at low-speed with flaps down. Never assume it is too slow to flutter.

2) Do not assume linkages are rigid and kinematics purely geometric.

3) Do a comprehensive GVT recording motion of all surfaces in all axes. Excite the ailerons (and maybe flaps) independently and over a range of amplitudes to reveal nonlinearities. Excite both in the antisymmetric and symmetric sense; the latter may be the more likely to cause problems.

4) A three-dimensional dynamic analysis to determine mass coupling is probably as useful as a full-blown flutter analysis. Ensure all mass couplings are leading-edge heavy over the expected amplitude range.

5) Although it is common practice in the industry that you show by analysis that it is free from flutter for any control surface frequency (up to 1.5 times measured value [12]), it is tempting fate to have an aileron with a nominal or measured rotation frequency close to a wing bending frequency, even if you think the mass coupling is favorable and flutter should not occur.

6) There should be no critical cases in narrow valleys of flutter speed for any parameters, or a strong dependence on parameters that cannot be easily varied or controlled. Then the FFT will be inconclusive, as the Nomad FFT demonstrated.

7) There should be no cases of positive feedback that can lead to explosive flutter; then FFT can be unacceptably dangerous. Any case of positive feedback, whether or not it is predicted flutter may occur in the flight speed range, should be regarded with great caution. It should be considered not good design.

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